1. Consider a planar manipulator with link lengths $r_1 = r_2 = 1$ and $r_3 = 1/2$. Find $\theta_1$, $\theta_2$, and $\theta_3$ such that the end effector is at (1,1) with orientation $\phi = -45^\circ$.

![Figure 1](image.png)

**Figure 1**: Mechanism of problem 1 but not in the configuration of the problem 1 statement.

2. We have seen a number of ways to describe the orientation of a rigid body with respect to a given coordinate frame. Consider the following rotation matrix:

$$
\begin{pmatrix}
\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\
\frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\
-\frac{1}{3} & \frac{2}{3} & \frac{2}{3}
\end{pmatrix}.
$$

Find the corresponding axis / angle parameters, the Euler-angles and the Tait-Bryan (pitch/roll/yaw) angles.

3. Which of the following subsets of $SE(3, \mathbb{R})$ are also subgroups?

(a) $\left\{ \begin{pmatrix} \cos \theta & 0 & \sin \theta & x \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & z \\ 0 & 0 & 0 & 1 \end{pmatrix} : x, z \in \mathbb{R}; -\pi < \theta \leq \pi \right\},$

(b) $\left\{ \begin{pmatrix} \cos \phi & -\sin \phi & 0 & 0 \\ \sin \phi & \cos \phi & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{pmatrix} : y, z \in \mathbb{R}; -\pi < \phi \leq \pi \right\}$

Justify your answer.
4. Prove that for any square matrices $A$ and $B$ that $e^{(A+B)t} = e^{At}e^{Bt} \iff [A, B] = 0$.

5. For the following $2 \times 2$ matrices, compute $e^{At}$:

(a) $A = \begin{pmatrix} \alpha & -\beta \\ \beta & \alpha \end{pmatrix}$

(b) $A = \begin{pmatrix} -a & a \\ b & -b \end{pmatrix}$