**ENG ME 740:**

Exercises (Set 4)

1. Write down the inverse of the matrix

\[
A = \begin{pmatrix}
\cos \theta & -\sin \theta & \cos \alpha & \sin \theta & \sin \alpha & a & \cos \theta \\
\sin \theta & \cos \theta & \cos \alpha & -\cos \theta & \sin \alpha & a & \sin \theta \\
0 & \sin \alpha & \cos \alpha & 0 & 0 & 0 & 1 \\
\end{pmatrix}
\]

3 pts

2. Given a prescribed position and orientation of the planar 3-bar manipulator of the second Exercise Set, there are two possible solutions to the inverse kinematics problem. If we add one more link (in such a way that the manipulator is still planar), how many solutions are there?

3 pts

3. The figure shows a 2-bar planar manipulator with rotary joints. The second link is half as long as the first \((r_1 = 2r_2)\). The joint limits are:

\[
0 < \theta_1 < 180^\circ \\
-90^\circ < \theta_2 < 180^\circ.
\]

4 pts

Sketch the approximate workspace (= the set of points which can be reached by the tip of the second link).
1. 
\[ A^{-1} = \begin{pmatrix} \cos \theta & \sin \theta & 0 & -a \\ -\sin \theta & \cos \theta & \cos \alpha & -d \sin \alpha \\ \sin \theta \sin \alpha & \cos \theta \sin \alpha & \cos \alpha & -d \cos \alpha \\ 0 & 0 & 0 & 1 \end{pmatrix} \]

(HINT: IF \((U \, v) \in SE(3, \mathbb{R})\), \((U \, v)^{-1} = (U^T \, -U^T v)\).)

2. **Infinitely Many.**

3. **See Next Pg.**
3. \[ r_1 = 2 \]
\[ r_2 = 1 \]

`endpointset = {}`

`Do[endpointset = Join[endpointset, \\
{\{r_1 * \text{Cos[}((j/40) \* \pi]) + r_2 * \text{Cos[}((j/40) \* \pi + (-\pi/2 + (k/80) \* \pi))]}, r_1 * \text{Sin[}((j/40) \* \pi]) + \\
r_2 * \text{Sin[}((j/40) \* \pi + (-\pi/2 + (k/80) \* \pi))])\}, {j, 1, 40}, {k, 1, 120}]`

`ListPlot[endpointset]`

The workspace looks like the shaded region.

The arc of radius \[ \sqrt{r_1^2 + r_2^2} \]