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(2/27/25)

ENG ME/SE 740:

Exercises (Set 3) (Due 3/6/25)

1. Prove that for any square matrices A and B that $e^{(A+B)t} = e^{At}e^{Bt} \iff [A, B] = 0$.
2. For the following 2×2 matrices, compute e^{At} :

$$(a) \ A = \begin{pmatrix} \alpha & -\beta \\ \beta & \alpha \end{pmatrix} \quad (b) \ A = \begin{pmatrix} -a & a \\ b & -b \end{pmatrix}$$

3. Find θ , ω and v such that

$$\text{Exp} \left[\begin{pmatrix} \hat{\omega} & v \\ 0 & 0 \end{pmatrix} \theta \right] = \begin{pmatrix} \frac{1+\sqrt{2}}{3} & \frac{2-\sqrt{2}(1+\sqrt{3})}{6} & \frac{2+\sqrt{2}(-1+\sqrt{3})}{6} & \frac{\sqrt{2}}{3} + \frac{\pi}{12} \\ \frac{2+\sqrt{2}(-1+\sqrt{3})}{6} & \frac{1+\sqrt{2}}{3} & \frac{2-\sqrt{2}(1+\sqrt{3})}{6} & \frac{-1}{3\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}} + \frac{\pi}{12} \\ \frac{2-\sqrt{2}(1+\sqrt{3})}{6} & \frac{2+\sqrt{2}(-1+\sqrt{3})}{6} & \frac{1+\sqrt{2}}{3} & \frac{-1}{3\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{6}} + \frac{\pi}{12} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$