Prof. J. Baillieul
Prof. T. Djaferis
Mechanical Engineering
Electrical and Computer Engineering

Solutions

(Do not use Matlab, Maple, Mathematica, etc. You may use Claude or ChatGPT, but say so if you do**)**

ENG ME 740:

Exercises (Set 4)

1. Write down the inverse of the matrix

3 pts

$$A = \begin{pmatrix} \cos\theta & -\sin\theta\ \cos\alpha & \sin\theta\ \sin\alpha & a\ \cos\theta \\ \sin\theta & \cos\theta\ \cos\alpha & -\cos\theta\ \sin\alpha & a\ \sin\theta \\ 0 & \sin\alpha & \cos\alpha & d \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

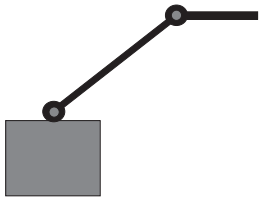
3 pts 2. Given a prescribed position and orientation of the planar 3-bar manipulator of the second Exercise Set, there are two possible solutions to the inverse kinematics problem. If we add one more link (in such a way that the manipulator is still planar), how many solutions are there?

4 pts 3. The figure shows a 2-bar planar manipulator with rotary joints. The second link is half as long as the first ($r_1 = 2r_2$). The joint limits are:

$$0 < \theta_1 < 180^\circ$$

$$-90^\circ < \theta_2 < 180^\circ.$$

Sketch the approximate workspace (= the set of points which can be reached by the tip of the second link).



EXERCISE SET 4 — SOLUTIONS — PROF. BAILLIEUL

MN 740

1.
$$A^{-1} = \begin{pmatrix} \cos\theta & \sin\theta & 0 & -a \\ -\sin\theta\cos\alpha & \cos\theta\cos\alpha & \sin\alpha & -d\sin\alpha \\ \sin\theta\sin\alpha & -\cos\theta\sin\alpha & \cos\alpha & -d\cos\alpha \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(HINT: IF $\begin{pmatrix} U & v \\ 0 & 1 \end{pmatrix} \in SE(3,\mathbb{R})$, $\begin{pmatrix} U & v \\ 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} U^T & -U^Tv \\ 0 & 1 \end{pmatrix}$.)

2. INFINITELY MANY.

3. SEE NEXT PG.

3.

```
r1 = 2
```

```
2
```

```
r2 = 1
```

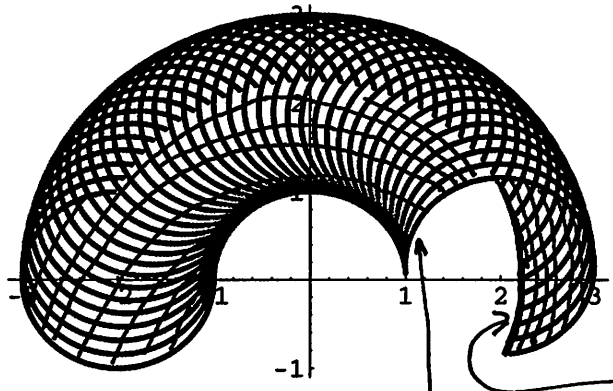
```
1
```

```
endpointset = {}
```

```
{}
```

```
Do[endpointset = Join[endpointset,
   {{r1 * Cos[(j / 40) * π] + r2 * Cos[(j / 40) * π + (-π / 2 + (k / 80) * π)], r1 * Sin[(j / 40) * π] +
      r2 * Sin[(j / 40) * π + (-π / 2 + (k / 80) * π)]}}], {j, 1, 40}, {k, 1, 120}]
```

```
ListPlot[endpointset]
```



- Graphics -

arc of radius $\sqrt{r_1^2 + r_2^2}$

arc of radius r_2

The workspace looks like the shaded region.