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(4/7/26)

ENG ME 740

Exercises (Set 5) (Due 4/14/26)

1. Consider the kinematics function associated with a *spherical wrist* mechanism.

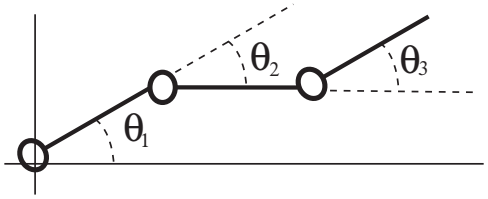
$$\begin{pmatrix}\theta_1\\ \theta_2\\ \theta_3\end{pmatrix} \mapsto \begin{pmatrix}\cos\theta_1 & -\sin\theta_1 & 0\\ \sin\theta_1 & \cos\theta_1 & 0\\ 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{pmatrix}\begin{pmatrix}\cos\theta_2 & -\sin\theta_2 & 0\\ \sin\theta_2 & \cos\theta_2 & 0\\ 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{pmatrix}$$
$$\cdot\begin{pmatrix}\cos\theta_3 & -\sin\theta_3 & 0\\ \sin\theta_3 & \cos\theta_3 & 0\\ 0 & 0 & 1\end{pmatrix}.$$

Compute the manipulator Jacobian (in the form of a 3×3 matrix) for this function. Sketch the mechanism, describe kinematically singular configurations in terms of your sketch, and explain the physical significance of these singularities.

2. A planar three link mechanism (See figure below.) is viewed as a (positionally) redundant manipulator. Assume the links all have equal length, and we assign (relative angle) coordinates θ_1, θ_2, and θ_3 in the usual way. Consider configurations which place the end-effector at prescribed (x, y)-coordinates so as to maximize the criterion $g(\theta) = \sin^2\theta_2 + \sin^2\theta_3$. The necessary condition for this placement to be optimal (with respect to maximizing g) is $\frac{\partial g}{\partial\theta}\cdot\vec{n} = 0$ where $\vec{n}$ is the (unit) null-space vector of the manipulator Jacobian.

(a) What configurations satisfy this necessary condition?

(b) What are the *algorithmic singularities* associated with this criterion?



3. The mechanism in the figure has four successive joints, with each joint axis $\perp$ to its predecessor and successor as shown.

(a) Write down the mechanism Jacobian as a 3×4 matrix.

(b) Sketch the kinematically singular configurations, and explain how velocities are physically restricted when the mechanism is in such a configuration.

(c) Explain why all singular configurations are avoidable.